Envelope-Finite Element (EVFE) Technique - 2-D Guided Wave Examples

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Abstract **— A novel full-wave technique, called EVFE method, is proposed to simulate the time-domain envelopes of electromagnetic waves. Based on finite element method (FEM) solutions, EVFE method introduces the circuit envelope simulation concept into electromagnetics the first time. Compared to traditional time-domain simulation techniques such as FDTD or FETD methods, only the signal envelope need to be sampled in EVFE simulation. Therefore it can bring magnitudes of computation savings when it is applied to the cases where signal envelope/carrier ratios are very small. In this paper, 2-D microwave guided-wave examples are presented as proof of concepts.**

I. INTRODUCTION

Traditional electromagnetic transient simulation techniques such as finite-difference time-domain (FDTD) methods or finite-element time-domain (FETD) methods have become very popular for the past two decades. Compared to their frequency domain predecessors, their capability to generate time-domain waveforms straightforwardly brings many advantages in simulating broadband system responses or identifying circuit parasitics [1]-[2]. They have also been developed for a co-simulation coupled with active/nonlinear devices in [3]-[4]. However, the time-step in simulation is usually required to be very small because of the CFL stability condition. Even when their implicit versions [5]-[6] are used, the time-domain waveform has to be sampled at minimum twice of the highest signal frequency to satisfy the Nyquist sampling criterion regardless of the signal bandwidth.

However, modern wireless and optical communication signals often employ digital modulations on the RF carrier or RF modulations on the optical carrier. The signal bandwidths in these systems are usually very narrow relative to their carrier frequencies. When transient simulators are used for this case, much of the computation is wasted. To address this limitation, a new circuit simulation technique called Circuit Envelope has been recently introduced in [7] and exploited in HPEEsof's ADS or MDS design software. By discretizing and simulating the signal envelopes on defined carrier frequencies, it has proven to be much more efficient than transient simulators like SPICE for narrow band cases.

Based on the similar concept, a novel electromagnetic solver called EVFE technique is proposed in this paper. Derived rigorously from Maxwell's equations, EVFE technique is able to simulate time-varying complex envelopes of electromagnetic waves. The essential idea is to perform time marching of the signal envelope on top of the frequency-domain FEM solutions. Since only the signal envelope needs to be sampled, much sparser time-step can be used than those in FDTD or FETD techniques, which results in much higher computation efficiency when the envelope/carrier ratio is small. Beside all the advantages as a full-wave time-domain technique, EVFE techniques also have considerable computational advantages over frequency-domain techniques like FEM, because there is no need to solve the boundary value problem once again for each different frequency. The formation and inversion of the finite element matrix only need to be done once for the defined carrier frequency, if a direct solver is used. The algorithm is also of low complexity and can be easily written by modifying a frequency-domain FEM code.

In this paper, for simplicity, only the EVFE techniques based on 2-D FEM is considered. There should not be any intrinsic limits to extend it for 3-D full-vector modeling of arbitrary guided-wave structures. This paper is organized as follows. The EVFE formulations are first derived from the scalar wave equations in chapter II, followed up by the implementation of traveling wave boundary conditions. Then two numerical examples are then presented in chapter III to validate the EVFE technique. Finally conclusion of this paper is given in chapter IV.

II. EVFE FORMULATIONS AND BOUNDARY CONDITIONS

Considering the 2-D TEM or TE wave propagating in a non-lossy planar waveguide, the time-domain wave equation regarding to longitudinal component of magnetic field is:

$$\frac{1}{\varepsilon_r \varepsilon_0}\nabla^2 H_z - \mu_r \mu_0 \frac{\partial^2 H_z}{\partial t^2} = 0 \qquad (1)$$

By defining the carrier frequency ω, the field component can be represented in a modulated signal format:

$$H_z(t) = V(t)e^{j\omega t} \qquad (2)$$

where *V(t)* is the time-varying complex envelope at the carrier frequency. It should be noted that the expression (2) is not unique but dependent on the definition of carrier frequency. Normally the carrier is chosen to be the center frequency of the interested frequency band in order to minimize the envelope frequency. Substituting (2) in (1) and dividing both sides by $e^{j\omega t}$ yields the partial differential equation (PDE) for the envelope:

$$\frac{1}{\varepsilon_r}\nabla^2 V(t) + K^2 \mu_r (1 - \frac{2j}{\omega}\frac{\partial}{\partial t} - \frac{1}{\omega^2}\frac{\partial^2}{\partial t^2}) \cdot V(t) = 0 \qquad (3)$$

where K is the free space wave number for the carrier frequency. ε_r and μ_r are the relative permittivity and permeability. Let's call (3) an envelope equation. It is noticed that the envelope equation reduces to scalar Helmholtz equation when V is time-independent, on which the frequency-domain FEM is based. On the other hand, if the carrier frequency ω is chosen to be zero, the envelope equation is also equivalent to the time-dependent wave equation on which the implicit FETD method is based. Therefore, one can easily solve it just like dealing with the other time-dependent wave equations. The inner product of (3) with a testing function T leads to the weak form

$$\iint_s \left[\frac{1}{\varepsilon_r}\nabla T \cdot \nabla V - K^2 \mu_r (1 - \frac{2j}{\omega}\frac{\partial}{\partial t} - \frac{1}{\omega^2}\frac{\partial^2}{\partial t^2}) \cdot T \cdot V \right] \cdot ds = \oint_\Gamma \frac{1}{\varepsilon_r} T \frac{\partial V}{\partial n} \cdot dl \qquad (4)$$

where the boundary Γ consists of PEC boundary Γ_c of the planar waveguide, excitation truncation boundary Γ_e and the termination truncation boundary Γ_t. Γ_c is natural boundary condition and has no contribution to the right-hand side path integral. For the truncation boundaries, the first-order absorbing boundary condition (ABC) is applied based on the traveling-wave assumption. The frequency-domain version of ABC is always a good approximation to use if the signal bandwidth is quite narrow. However, rigorous implementation of ABC in EVFE simulation requires special handling as follows. First let's consider the termination boundary Γ_t. The traveling wave assumption in time-domain is

$$\frac{\partial H_z}{\partial x} + \frac{1}{c}\frac{\partial H_z}{\partial t} = 0 \qquad (5)$$

where c is the free space light speed. Substituting (2) in (5) leads to the envelope boundary condition

$$\frac{\partial V(t)}{\partial x} = -\frac{1}{c}\frac{\partial V(t)}{\partial t} - j\frac{\omega}{c}V(t) \qquad (6)$$

The boundary condition for the excitation can be derived in the similar way

$$\frac{\partial V(t)}{\partial x} = \frac{1}{c}\frac{\partial V(t)}{\partial t} + \frac{j\omega}{c}V(t) - \frac{2}{c}\frac{\partial V^i(t)}{\partial t} - \frac{2j\omega}{c}V^i(t) \qquad (7)$$

where V^i is the envelope of incident field. (6) and (7) should be substituted into the right-hand side path integral of (4). Expanding the envelope variable using 2-D FEM basis functions W_j, the application of Galerkin's process results in a system of ordinary differential equations

$$[T]\frac{d^2 v}{dt^2} + [B]\frac{dv}{dt} + [S]v + f = 0 \qquad (8)$$

where v is the coefficient vector of V and *[T], [B]* and *[S]* are time-independent matrics defined by

$$\begin{aligned} T_{ij} &= \iint_s \frac{\mu_r}{c^2} W_i W_j ds \\ B_{ij} &= \iint_s \frac{2j\omega\mu_r}{c^2} W_i W_j ds + \oint_\Gamma \frac{1}{c\varepsilon_r} W_i W_j dl \\ S_{ij} &= \iint_s \frac{1}{\varepsilon_r}\nabla W_i \cdot \nabla W_j - K^2 \mu_r W_i W_j ds + \oint_\Gamma \frac{j\omega}{c\varepsilon_r} W_i W_j dl \\ fi &= \oint_{\Gamma_e} \left[-\frac{2}{c\varepsilon_r}\frac{\partial v^i}{\partial t} - \frac{2j\omega}{c\varepsilon_r} v^i \right] \cdot W_i W_j dl \end{aligned} \qquad (9)$$

To discretize (8) in time-domain, the Newmark-Beta formulation [6] can be used

$$\begin{cases} \dfrac{d^2 v}{dt^2} = \dfrac{1}{\Delta t^2}\left[v(n+1) - 2v(n) + v(n-1)\right] \\ \dfrac{dv}{dt} = \dfrac{1}{2\Delta t}\left[v(n+1) - v(n-1)\right] \\ v = \beta v(n+1) + (1-2\beta)v(n) + \beta v(n-1) \end{cases} \quad (10)$$

where *v(n) = v(nΔt)* is the discrete-time representation of *v(t)*. β is a constant. Similar to what is done for FETD method in [5]-[6], one can prove that $\beta=1/4$ leads to an unconditionally stable two-step update scheme with minimum dispersion error also for EVFE technique, which is

$$\left[\frac{[T]}{\Delta t^2} + \frac{[B]}{2\Delta t} + \frac{[S]}{4}\right]v(n+1) = \left[\frac{2[T]}{\Delta t^2} - \frac{[S]}{2}\right]v(n) + \left[-\frac{[T]}{\Delta t^2} + \frac{[B]}{2\Delta t} - \frac{[S]}{4}\right]v(n-1) - f(n) \quad (11)$$

To solve the above equations, the matrix in the left-hand side needs to be inversed. Note this matrix is time-independent, it needs to be filled and solved only once if a direct sparse-matrix solver is used.

III. Numerical Results

Two numerical examples are presented to validate the above formulations. The first example is an empty planar waveguide, depicted in Fig.1. The waveguide is 6 meter long and 4 cm wide. Incident wave is a modulated Gaussian pulse, in the form of

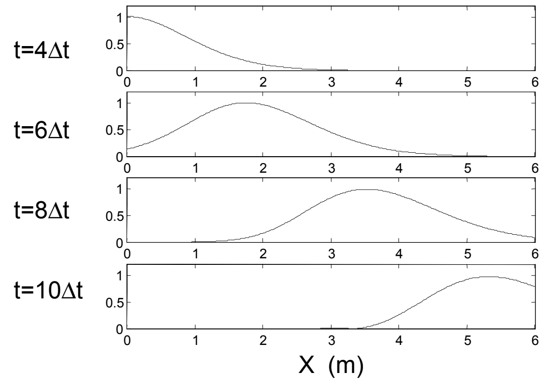

Fig.2 The magnetic field envelope along the waveguide for different observation time.

$$H_z^i(t) = \exp\left[\frac{(t-3\sigma_t)^2}{2\sigma_t^2}\right] \bullet e^{j\omega t} \quad (12)$$

where $\omega = 2\pi f, f = 2.91GHz$, $\sigma_t = 3.0ns$. For this excitation, the envelope/carrier frequency ratio is about 10 percent. Therefore a time-step $\Delta t = 1.5ns$ is used for the EVFE simulation, which is much wider than the carrier wavelength. The total number of time steps used is 16. To get the same precision, an explicit FDTD method needs a number of time steps at least 100 times of that and an implicit FETD simulation needs at least 10 times of that. Since there is no discontinuity, the electromagnetic wave should propagate without dispersion. Fig.2 plots the field envelope along the waveguide at different time steps, where a nice traveling wave effect is observed.

The second example is again the same planar waveguide, but with two double irises in the middle. As depicted in Fig.3, the length of the iris is 1.5 cm. The two apertures are constructed 5 cm away from each other, to intentionally form a resonance peak around the carrier frequency. The same excitation and time-step as

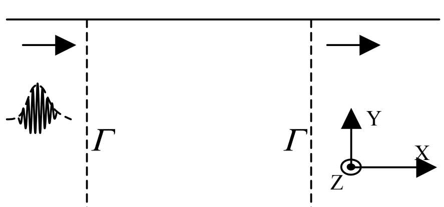

Fig.1 Sketch of an empty planar waveguide.

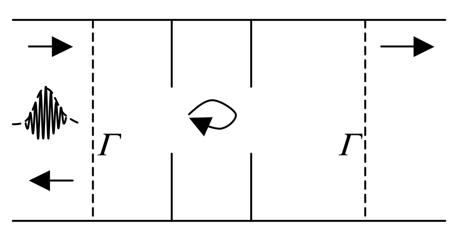

Fig.3 Sketch of an planar waveguide with two double irises.

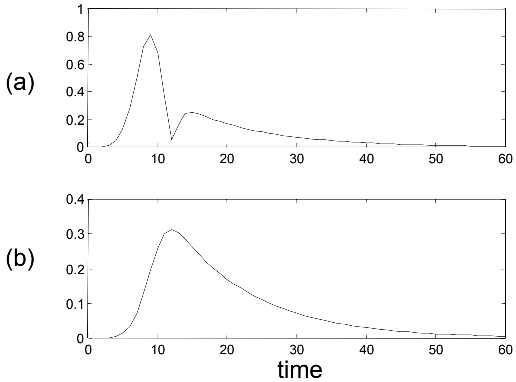

Fig.4 Time domain waveform for (a) return wave (b) through wave.

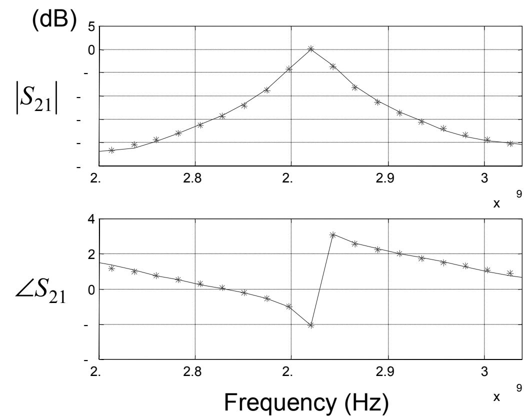

Fig.6 Comparison of S_{21} between EVFE result (--) and FEM result (*).

in the first example are used. After the simulation, the time-domain waveform is recorded in both the excitation plane and the termination plane. As shown in Fig.3, the long tails of the signals are observed in both return wave and through wave, which indicates a sharp frequency resonance. The total number of time steps is 60. By applying Fourier transform to the time-domain waveform, the S parameters are generated and plotted in Fig.4 and Fig.5 against the frequency-domain FEM results. Very good agreements can be seen from those comparisons, which further validates the approach.

IV. Conclusion

A novel full-wave electromagnetic simulation method called EVFE technique has been proposed based on the envelope simulation concept. When applied to the cases with the slowly varying signal envelope on top of fast oscillating carrier, it can provide magnitudes of computation saving over the traditional time-domain techniques. Two 2-D numerical examples have been presented to validate the approach. It can also be considered as a more general electromagnetic simulation frame that unites the frequency-domain and time-domain techniques, since it reduces to frequency-domain FEM when the envelope is constant and to FETD when the carrier frequency is chosen to be zero.

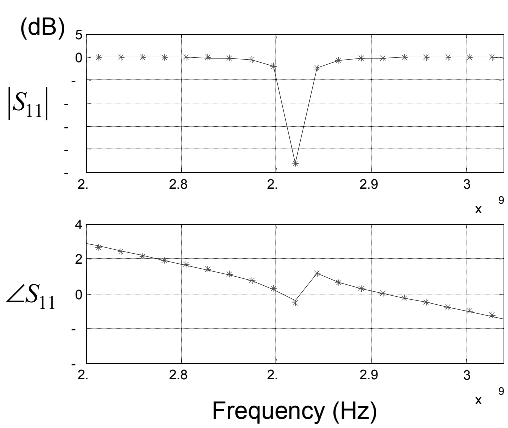

Fig.5 Comparison of S_{11} between EVFE result (--) and FEM result (*).

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